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ADAPTIVE EXCITATION FOR SELECTIVE SENSITIVITY-BASED STRUCTURAL IDENTIFICATION

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Abstract. *Major problems of applying selective sensitivity to system identification are requirement of precise knowledge about the system parameters and realization of the required system of forces. This work presents a procedure which is able to deriving selectively sensitive excitations by iterative experiment. The method first uses a priori information to determine force and displacement patterns with respect to the selected parameters. Agreement between actual forces and the calculated values is ensured by a predictive control algorithm. Parameter updating is then done by minimizing the difference between a predicted displacement response and the selectively sensitive displacement. These updated values become again prior information in the next test and the experiment is performed until a close match between model ouput and measured output can be achieved. As an illustration a simply supported beam made of steel, vibrated by harmonic excitation is investigated, thereby demonstrating that the adaptive excitation can be obtained efficiently.*

1 INTRODUCTION

A critical issue in system identification is the treatment of ill-conditioned, noisy mathematical formulations, e.g. linear equations. Such equations often arise in the estimation of parameter values for a given structural model by using vibration measurements. The consequence of the ill-conditionedness is that any small levels of measurement noise may lead to a large deviation in the identified parameters from their exact values [1]. The problem tends to become more pronounced as the number of parameters increases.

One solution to the problem of ill-conditioning is to design a system of excitation forces which produces strong sensitivities to a small number of parameters selected for identifying while causing the sensitivities to other parameters to vanish. This approach is referred to as the method of selective sensitivity which was introduced in [2, 3]. The aim is to adapt the load system so that the output is sensitive to the selected parameters and insensitive to others, thus transforming the original (large) identification problem into a sequence of smaller ones. One major disadvantage of this strategy is the requirement of fairly good knowledge of all parameters to be tested. Some efforts have been made to overcome this difficulty. Cogan *et al.* [4] used the modal model to construct the forces. Bucher and Pham [5] presented a new approach which requires no prior information of the parameters, however, is appropriate only for statically determinate structures and applicable when the frequency of excitation is kept below the fundamental frequency of the system. Generally, in order to obtain such adaptive excitations, it is necessary to set up an iterative experiment. On the other hand, the physical difficulty of applying the required system of forces, which often relatively large and possibly complex, poses a serious problem to practical application [6]. For instance, momental excitations are experimentally rather difficult to realize, or it is probably difficult to ensure that the actual forces on the tested structure will agree with the input energies (excitation signals) based on the calculated force patterns. In this case, an alternative force configuration and a force control are desirable. Some researches have addressed to this problem with certain limitations [7, 8].

This paper presents a new iterative procedure to obtain selectively sensitive excitations for dynamic identification of linear, undamped structures. The first step is to determine the selectively sensitive displacement and selectively sensitive force patterns. These values are obtained by introducing the prior information of system parameters into an optimization which minimizes the sensitivities of the structure response with respect to the unselected parameters while keeping the sensitivities with respect to the selected parameters as a constant. In a second step the force pattern is used to derive dynamic loads on the tested structure and measurements are carried out. An automatical control ensures the required excitation forces. In a third step, measured outputs are employed to update the prior information. The strategy is to minimize the difference between a predicted displacement response, formulated as function of the unknown selected parameters and the measured displacements, and the selectively sensitive displacement calculated in the first step. With the updated values of the parameters a re-analysis of selective sensitivity is performed and the experiment is repeated until the displacement responses of model and actual structure are conformed. The general concept is shown in Figure 1. The feasibility of the proposed procedure is demonstrated by a laboratory experiment.

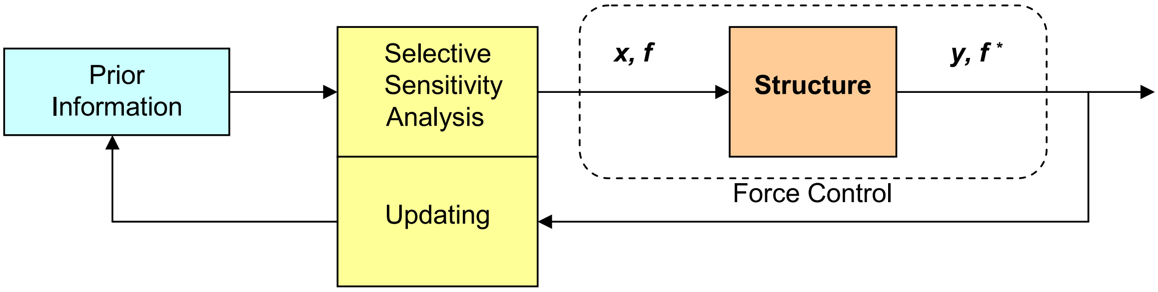

Figure 1: Schematic representation of the exterimental procedure

2 METHOD OF ANALYSIS

2.1 Selective sensitivity and adaptive excitation

Consider the case when tests are performed using low-amplitude vibrations so that the non-linear behavior can be neglected. The structure is then modelled as a N_d degrees of freedom (DOFs) undamped, linear system governed by the equation of motion

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \tag{1}$$

in which $\mathbf{M}$ denotes the mass matrix, $\mathbf{K}$ is the stiffness matrix, $\mathbf{f}$ is the external excitation, and $\mathbf{x}$ is the displacement response. In the frequency domain, the equation becomes

$$\mathbf{S}(\omega)\mathbf{X}(\omega) = \left(-\omega^2\mathbf{M} + \mathbf{K}\right)\mathbf{X}(\omega) = \mathbf{F}(\omega) \tag{2}$$

with $\mathbf{X}$ and $\mathbf{F}$ are the Fourier-transforms of $\mathbf{x}$ and $\mathbf{f}$, respectively. Assume that the mass matrix is known with sufficient accuracy whereas the stiffness matrix is uncertain and parameterized by

$$\mathbf{K} = \mathbf{K}_0 + \sum_{p=1}^{N_p} \theta_p \mathbf{K}_p \tag{3}$$

in which $\mathbf{K}_0$ is the nominal stiffness matrix and the unknown parameters θ_p have to be determined from an identification procedure. Typically, $\mathbf{K}_p$ represent the given substructure matrices defining location and type of parameter uncertainties.

According to the concept of selective sensitivity, if a displacement vector $\mathbf{X}_s$ can be found, so that it satisfies the condition

$$\begin{aligned} \mathbf{K}_p\mathbf{X}_s &= \mathbf{0} \text{ for } p \neq s \\ \mathbf{K}_p\mathbf{X}_s &\neq \mathbf{0} \text{ for } p = s \end{aligned} \tag{4}$$

then an excitation $\mathbf{F}_s$ (named as selectively sensitive excitation) computed from

$$\mathbf{F}_s = \left(-\omega^2\mathbf{M} + \mathbf{K}_0 + \sum_{p=1}^{N_p} \theta_p \mathbf{K}_p\right)\mathbf{X}_s = \mathbf{F}_s^{(0)} + \triangle\mathbf{F}_s \tag{5}$$

where

$$\mathbf{F}_s^{(0)} = \left(-\omega^2\mathbf{M} + \mathbf{K}_0\right)\mathbf{X}_s; \quad \triangle\mathbf{F}_s = \sum_{p=1}^{N_p} \theta_p\mathbf{K}_p\mathbf{X}_s = \theta_s\mathbf{K}_s\mathbf{X}_s \tag{6}$$

will cause the system output $\mathbf{Y}$ to depend only on the selected parameters θ_s, i.e,

$$\mathbf{Y} = \mathbf{G}\left(-\omega^2\mathbf{M} + \mathbf{K}_0 + \sum_{p=1}^{N_p}\theta_p\mathbf{K}_p\right)^{-1}\mathbf{F}_s = \mathbf{G}\left(-\omega^2\mathbf{M} + \mathbf{K}_0 + \theta_s\mathbf{K}_s\right)^{-1}\left(\mathbf{F}_s^{(0)} + \triangle\mathbf{F}_s\right) \quad (7)$$

However, it is usually not possible to apply excitations at all DOFs in practice, especially for rotational DOFs. Commonly, alternative force configurations are chosen, i.e. the excitation vector is given by $\mathbf{Tf}(t)$. Here $\mathbf{T}$ and $\mathbf{G}$ are rectangular matrices which locate the loads and outputs, respectively. The equation of motion becomes

$$\mathbf{S}(\omega)\mathbf{X}(\omega) = \left(-\omega^2\mathbf{M} + \mathbf{K}\right)\mathbf{X}(\omega) = \mathbf{TF}(\omega) \quad (8)$$

and the output $\mathbf{Y}$ is obtained from the displacement response through

$$\mathbf{Y} = \mathbf{G}\cdot\mathbf{X} = \mathbf{GS}^{-1}\mathbf{TF} \quad (9)$$

Its sensitivity S_p with respect to θ_p is

$$S_p = \left(\frac{\partial\mathbf{Y}}{\partial\theta_p}\right)^T\left(\frac{\partial\mathbf{Y}}{\partial\theta_p}\right) = \mathbf{F}^T\mathbf{T}^T\left(\frac{\partial\mathbf{S}^{-1}}{\partial\theta_p}\right)^T\mathbf{G}^T\mathbf{G}\left(\frac{\partial\mathbf{S}^{-1}}{\partial\theta_p}\right)\mathbf{TF} \quad (10)$$

The derivative of the matrix $\mathbf{S}^{-1}$ is computed from

$$\frac{\partial\mathbf{S}^{-1}}{\partial\theta_p} = -\mathbf{S}^{-1}\frac{\partial\mathbf{S}}{\partial\theta_p}\mathbf{S}^{-1} = -\mathbf{S}^{-1}\mathbf{K}_p\mathbf{S}^{-1} \quad (11)$$

which leads to

$$S_p = \mathbf{F}^T\mathbf{D}_p\mathbf{F} \quad (12)$$

$$\mathbf{D}_p = \mathbf{T}^T\left(-\mathbf{S}^{-1}\mathbf{K}_p\mathbf{S}^{-1}\right)^T\mathbf{G}^T\mathbf{G}\left(-\mathbf{S}^{-1}\mathbf{K}_p\mathbf{S}^{-1}\right)\mathbf{T} \quad (13)$$

The purpose of selective sensitivity is to provide excitation vectors $\mathbf{F}_s$ in such a way, that the sensitivities of measured output to change in the parameters θ_p become (almost) zero for all $p \neq s$. These excitations should meet the (weakly) selectively sensitive condition

$$\begin{aligned}\mathbf{F}_s^T\mathbf{D}_p\mathbf{F}_s &= small \text{ for } p \neq s \\ \mathbf{F}_s^T\mathbf{D}_p\mathbf{F}_s &= large \text{ for } p = s\end{aligned} \quad (14)$$

One possibility to obtain $\mathbf{F}_s$ is to minimize the sensitivities with respect to the parameters $\theta_p(p \neq s)$, while maintaining the sensitivities with respect to the parameters θ_s as a constant, i.e $S_s = \alpha_s$. The objective function for this optimization problem therefore is defined as

$$J_s = \sum_p \mathbf{F}_s^T\mathbf{D}_p\mathbf{F}_s + \lambda\left(\mathbf{F}_s^T\mathbf{D}_s\mathbf{F}_s - \alpha_s\right) \quad (15)$$

where λ denotes a Lagrange multiplier. The condition to obtain optimum is $\frac{\partial J_s}{\partial\mathbf{F}_s} = 0$ which leads to the eigenproblem

$$\sum_p \mathbf{D}_p\mathbf{F}_s = -\lambda\mathbf{D}_s\mathbf{F}_s \quad (16)$$

in which $-\lambda$ is the eigenvalue and $\mathbf{F}_s$ is the corresponding eigenvector. The objective is to minimize J_s, thus the smallest eigenvalue and its corresponding eigenvector are selected. The displacement response $\mathbf{X}_s$ caused by $\mathbf{F}_s$, is then determined from

$$\mathbf{X}_s = \left(-\omega^2\mathbf{M} + \mathbf{K}\right)^{-1}\mathbf{TF}_s \quad (17)$$

2.2 Updating paramter values and excitation forces

Since computing such adaptive excitation, however, involves the knowledge of the parameters to be identified, an iterative experiment procedure with respect to the selected parameters θ_s is suggested.

This iterative procedure is based on the concept of predictive control. The idea is to provide a suitable control force that minimizes the difference between system response prediction and a reference trajectory, which is $\mathbf{X}_s$ in this case. Suppose that the unknown parameters θ_s can be updated after N_t tests, $\theta_s = \sum_{i=1}^{N_t} \triangle\theta_s^{(i)}$, the required control force for the test $(t+1)$–th, denoted by $\triangle\mathbf{F}_s^{(t+1)}$, can be written in the form

$$\triangle\mathbf{F}_s^{(t+1)} = \triangle\theta_s^{(t+1)}\mathbf{K}_s\mathbf{X}_s \tag{18}$$

By noticing Eq. 7, the predicted system output is formulated as following

$$\mathbf{Y}^{(t+1)} = \widehat{\mathbf{Y}}^{(t)} + \mathbf{G}\left(-\omega^2\mathbf{M} + \mathbf{K}^{(t)} + \triangle\theta_s^{(t+1)}\mathbf{K}_s\right)^{-1}\triangle\theta_s^{(t+1)}\mathbf{K}_s\mathbf{X}_s \tag{19}$$

$$\mathbf{K}^{(t)} = \mathbf{K}_0 + \sum_{i=1}^{t}\triangle\theta_s^{(i)}\mathbf{K}_s \tag{20}$$

where $\widehat{\mathbf{Y}}^{(t)}$ is the measured output at test t–th under the excitation force $\mathbf{F}_s^{(t)}$. Thus, these predictions are function of the unknowns $\triangle\theta_s^{(t+1)}$ and the current state of the system. The values $\triangle\theta_s^{(t+1)}$ are determined by minimizing

$$P = \parallel \mathbf{Y}^{(t+1)} - \mathbf{G}\mathbf{X}_s \parallel^2 \tag{21}$$

With the updated values of the parameters θ_s, the excitation force is then recomputed by solving the eigenproblem Eq. 16.

Here, the required forces are obtained indirectly. Clearly, the robustness of this procedure largely depends on the accuracy of the applied excitation forces on the tested structure. In most cases, a force controlling algorithm is necessary to ensure selective sensitivity. Basis of such algorithm is, again, predictive control, which is explained in the next section.

3 AUTOMATICAL FORCE CONTROL

To excite the structure into vibration, signals are generated and transferred to the shakers attached on the structure. These signals in the form of voltage will be amplified in order to drive the actual devices. For selectively sensitive forces, it is reasonable to use excitation signals which are sinusoidal. Thus, a vector of signal amplitudes (vector of input voltages) will be sufficient for generating. This vector is supposed to be proportional to the required forces, but it is commonly not. Moreover, the phases of the actual forces may be far from expectation. In this case, predictive control will be used.

Let **v** and **p** are vectors of input voltage and input phase, respectively. Our task is to adjust these values so that the required excitation forces can be achieved, i.e.

$$\mathbf{v} = \mathbf{v}_0 + \triangle\mathbf{v};\ \mathbf{p} = \mathbf{p}_0 + \triangle\mathbf{p}; \tag{22}$$

where $\triangle\mathbf{v}$ and $\triangle\mathbf{p}$ are, respectively, the control values of voltage and phase and will be determined by minimizing the error between the predicted and the required values of force and phase. The formulation of this error is given by

$$error = \| (\mathbf{F}_s^* + \mathbf{A}\triangle\mathbf{v}) - \mathbf{F}_s \|^2 + \| (\mathbf{p}_s^* + \mathbf{B}\triangle\mathbf{p}) - \mathbf{p}_s \|^2 + w_v \| \triangle\mathbf{v} \|^2 + w_p \| \triangle\mathbf{p} \|^2 \quad (23)$$

where $\mathbf{F}_s^*$ and $\mathbf{p}_s^*$ are, respectively, vectors of measured forces and measured phases corresponding to the initial inputs $(\mathbf{v}_0, \mathbf{p}_0)$; **A** and **B** are constant matrices referred to as *internal model* of the control system; w_1 and w_2 are the weights put on the changes in force and phase inputs. The reason for introducing the two terms $w_v \| \triangle\mathbf{v} \|^2$ and $w_p \| \triangle\mathbf{p} \|^2$ is that the changes in the input signals are unwanted.

The control will be processed untill measured values meet the requirement. Obviously, this process depends on the choice of the matrices **A** and **B** and the weights w_1 and w_2. A simple choice is to set **A** and **B** equal to the identity matrix. For optimal cotrol a *trial and error* procedure prior to actual tests is suggested.

4 LABORATORY EXPERIMENT

As an illustration of the proposed procedure a laboratory experiment is carried out. The objective is to identify the bending stiffness of a simply supported beam modelled as 4-beam-element structure. Dynamic excitations are realized by using controlable shakers attached on the beam.

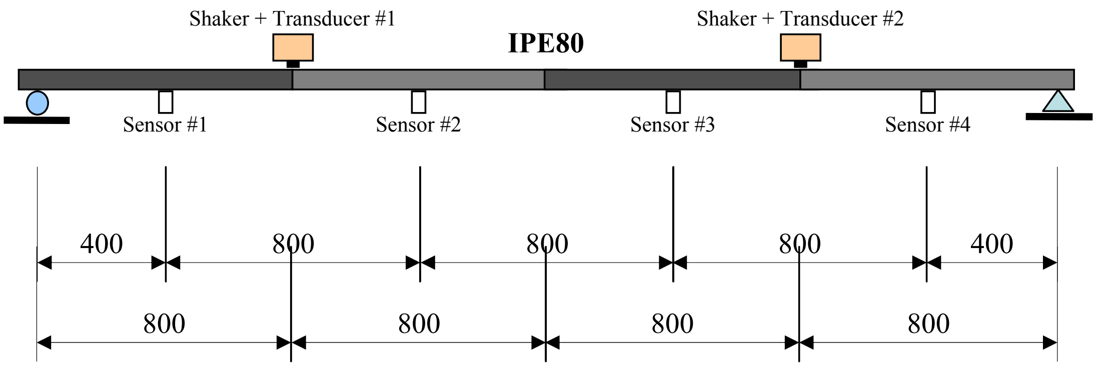

Figure 2: Experiment model

4.1 Experiment set-up and data analyzing

Figure 2 shows the layout of the experiment model. The beam is made from a standard steel profile IPE80 and is designed so that its local bending stiffness can be easily changed. The softwares are developed in the programming environment LabVIEW so that all of the analyzing, shaking and measuring modules are combined in a unique program. The hardwares used are a personal computer (Pentium 4 processor, Windows XP) with a National Instruments PCI-6024E Data Acquisition Card and a BNC-2120 Connector. The shaking and measuring devices include two low bass shakers, four accelerometers KB12V, two force transducers (M) 201B01, a 150W Sony stereo amplifier, signal conditioners, cables.

All data measured in time-series are transformed into frequency-domain by using Fast Fourier Transformation (FFT). Displacements are obtained by integrating the acquired accelerations twice. Since the vibration is harmonic and measurements are made in steady state, measured values to be used are picked up only at the frequency of excitation. The following part shows the test results from exciting the structure at two different frequencies.

4.2 Test results

Let $EI_p;\ p = 1 \ldots 4$ denote the bending stiffness of the four elements. Note that selectively sensitive forces produce stresses only in the elements whose stiffness is to be identified [5]. For our case, analyses show that selective sensitivity can be *best* achieved for the following parameter selections: $(EI_1\ and\ EI_2)$ or $(EI_3\ and\ EI_4)$, i.e at least two parameters. An example of selectively sensitive displacement patterns with respect to parameter pair $(EI_1\ and\ EI_2)$ clearly indicates (almost) no stress in the unselected elements (see Figure 3).

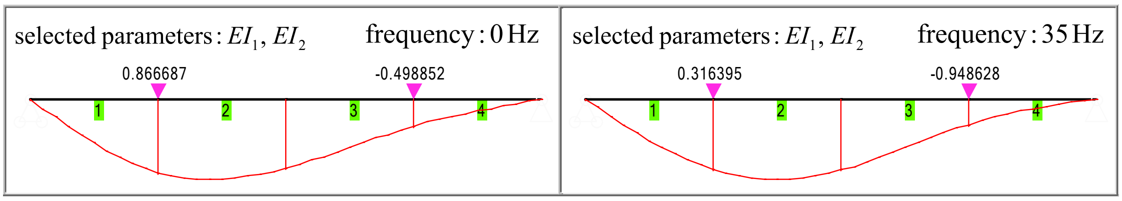

Figure 3: Displacement patterns with respect to parameter pair $(EI_1\ and\ EI_2)$

One of our tasks is to choose the suitble frequencies for excitation. In principle, frequencies of all ranges can be used. In fact, low-frequency vibration may easily be dominatated by the first mode, making selective sensitivity difficult to obtain. On the other hand, the influence of mass will become considerable when high excitation frequencies are appied. A pre-testing shows the first natural frequency of the structure to be approximately 23Hz and approximately 91Hz for the second one. In this experiment, frequencies in the range [30Hz, 39Hz], i.e. higher the first and lower the second, have been found to be useful.

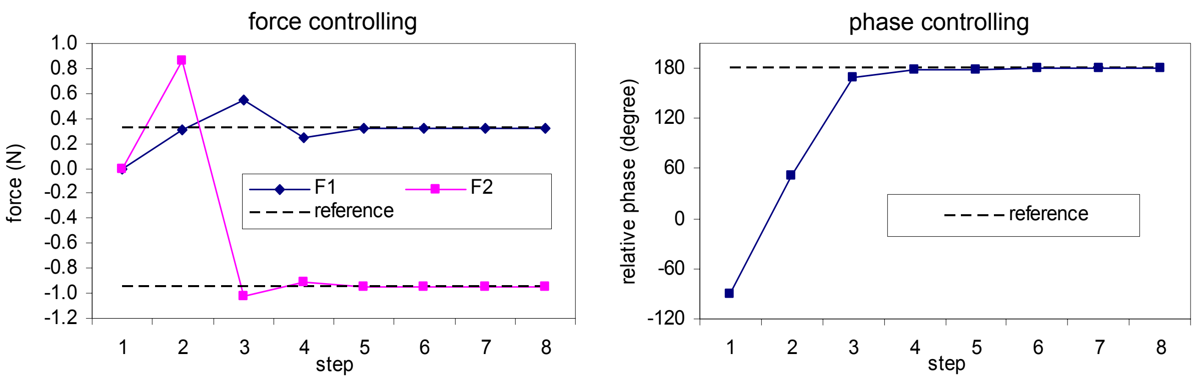

Figure 4: Results of controlling excitation forces of 35Hz in test 1

Setting the initial bending stiffness values of all elements equal to $EI_0;\ E = 2.1{\times}10^{11} N/m^2$,

$I_0 = 8.0 \times 10^{-7} m^4$, the updated values of $I_p;\ p = 1 \ldots 4$ using excitation frequencies of 35Hz and 37Hz are listed in Table 1 and Table 2. Herein, at each updating step two parameters are chosen for dynamic testing. These parameters have values appear in the tables, e.g at test 1 and test 2 are I_1 and I_2; at test 3 and test 4 are I_3 and I_4 and so on. The calculated force vectors and the actual forces are also presented (see Table 1 and Table 2). These forces are well controlled to the required values (reference values) in some steps (see Figure 4). In addition, the errors (in percentage) between model and measured displacements are viewed for each test, which show that a close match between model outputs and measured outputs can be achieved after some few tests (see Table 1 and Table 2).

35Hz	Force (N)		Bending stiffness $(\times 10^{-7} m^4)$				Displacement error
test	calculated	measured	I_1	I_2	I_3	I_4	$\frac{\lVert\widehat{\mathbf{Y}}-\mathbf{G}\mathbf{X}_s\rVert}{\lVert\widehat{\mathbf{Y}}\rVert} \times 100$
0			8.0	8.0	8.0	8.0	
1	0.316; -0.949	0.317; -0.947	8.368	10.19			12.296
2	0.444; -0.896	0.444; -0.896	8.580	10.00			1.091
3	0.957; -0.290	0.957; -0.290			8.847	9.299	9.061
4	0.927; -0.374	0.928; -0.373			8.927	9.185	2.763
5	0.443; -0.897	0.443; -0.896	8.952	9.750			2.037

Table 1: Test results with excitation frequency 35Hz

37Hz	Force (N)		Bending stiffness $(\times 10^{-7} m^4)$				Displacement error
test	calculated	measured	I_1	I_2	I_3	I_4	$\frac{\lVert\widehat{\mathbf{Y}}-\mathbf{G}\mathbf{X}_s\rVert}{\lVert\widehat{\mathbf{Y}}\rVert} \times 100$
0			8.0	8.0	8.0	8.0	
1	0.224; -0.975	0.225; -0.975	8.694	9.832			9.986
2	0.361; -0.933	0.361; -0.933	9.200	9.539			1.741
3	0.975; -0.223	0.975; -0.223			8.850	9.466	8.045
4	0.949; -0.316	0.949; -0.317			8.899	9.474	2.129
5	0.948; -0.319	0.948; -0.319			8.914	9.466	2.100
6	0.365; -0.931	0.365; -0.931	8.772	9.737			0.940
7	0.367; -0.930	0.366; -0.931	8.752	9.794			0.887

Table 2: Test results with excitation frequency 37Hz

As can be seen from the above experimental results, the proposed procedure allows to obtain quite efficiently the adaptive excittions required to identify the selected parameters of the structure under investigation.

5 CONCLUSIONS

An iteratively experimental procedure to obtain selectively sensitive excitations for dynamic identification of linear, undamped structures has been proposed. By means of predictive control this approach provides an efficient tool to derive the required excitation forces and to update the selected parameters, simultaneously. In this way, the method of selective sensitivity becomes feasible and useful for reducing ill-conditionedness in system identification.

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